

All children must be able to use these methods before moving on to other methods

- plus
- increase
- make
- together

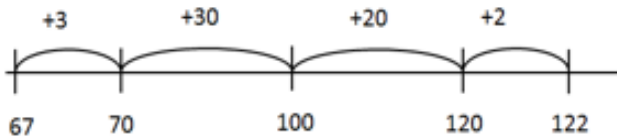
Using Place value

Count in thousands, e.g. knowing $475 + 200$ as 475, 575, 675

Partitioning, e.g. $746 + 203$ as $700 + 200$ and $46 + 3$
or $134 + 707$ as $130 + 700$ and $4 + 7$

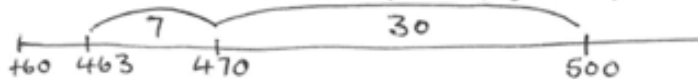
Counting on

To jump to the next multiple of ten – reinforce the use of number bonds



Using number facts

Number bonds to 100 and to next multiple of 100, e.g. $463 + 37$, $1353 + 47$



Number bonds to £1 and to the next whole pound, e.g. $£3.45 + 55p$

Add to next whole number, e.g. $4.6 + 0.4$, $7.2 + 0.8$

Build on partitioning to develop expanded column addition with two 3-digit numbers $466 + 358 =$

| | | |
|------|-----|----|
| 400 | 60 | 6 |
| +300 | 50 | 8 |
| | | |
| 700 | 110 | 14 |

- Start by adding the ones

Expanded column addition with 'carrying'

Compact column addition with two or more 3-digit numbers or towers of 2-digit numbers

| | | |
|------|----|---|
| 400 | 60 | 6 |
| +300 | 50 | 8 |
| | | |
| 100 | 10 | |
| | | |
| 800 | 20 | 4 |

- Start by adding the ones

Build on expanded column addition to develop compact column addition

| | | | |
|-------|-----|----|---|
| 1000 | 400 | 60 | 6 |
| +4000 | 800 | 60 | 8 |

Year 3

Year 4

Year 5

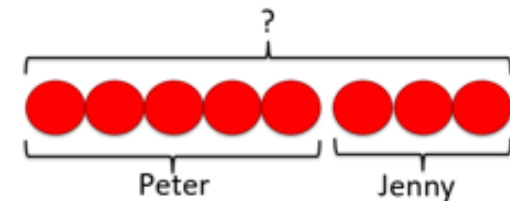
Peter has 5 cakes and Jenny has 3 cakes. How many cakes do they have altogether?

Concrete/Pictorial Model



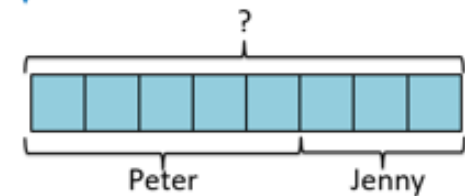
The demonstration of concrete

Pictorial Model



Students can draw a picture

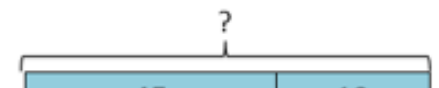
Discrete Bar Model



Using square blocks to represent

Class 1 have 45 pencils and Class 2 have 30 pencils. How many pencils do they have altogether?

Fully Continuous Model



Year 6

Compact column addition with larger numbers.
Use expanded and compact column addition to add amounts of money.

$$\begin{array}{r} 5347 \\ 2286 \\ +1495 \\ \hline 9128 \end{array}$$

Expanded column addition for money leading to compact column addition for adding several amounts of money
Compact column addition to add Pairs of 5-digit numbers

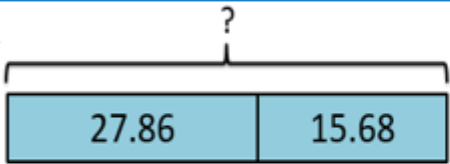
$$\begin{array}{r} \text{£}14 \text{ 60p 4p} \\ \text{£}28 \text{ 70p 8p} \\ + \text{£}12 \text{ 20p 6p} \\ \hline \text{£}1 \text{ 10p} \\ \hline \text{£}55 \text{ 60p 8p} \end{array}$$

Continue to use column addition to add towers of several larger numbers.
Use compact addition to add decimal numbers with up to two places

$$27.86 + 15.68 =$$

$$\begin{array}{r} 15.68 \\ +27.86 \\ \hline 11.1 \\ \hline 43.54 \end{array}$$

The length of Jenny's hair was 27.86cm in March. Since t has grown by 15.68cm. How long is it now?

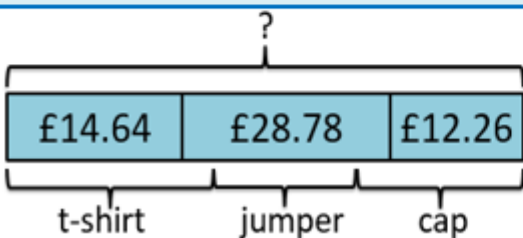


Compact column addition for adding several large numbers and decimal numbers with up to two places

$$\text{£}14.64 + \text{£}28.78 + \text{£}12.26 =$$

$$\begin{array}{r} \text{£}14.64 \\ + \text{£}28.78 \\ \text{£}12.26 \\ \hline 11.1 \\ \hline \text{£}55.68 \end{array}$$

Peter buys a t-shirt for £14.64, a jumper for £28.78 and a for £12.26. How much did his shopping cost altogether?



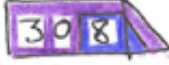


Question mark to label what you are looking to solve.

All children must be able to use these methods before moving on to other strategies

Year 3

Subtract near multiples, e.g. $648 - 199$ or $86 - 39$

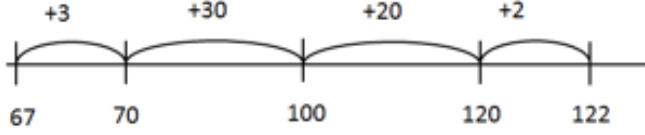
Counting up  -  

Find a difference between two numbers by counting up from the smaller to the larger, e.g. $121 - 87$

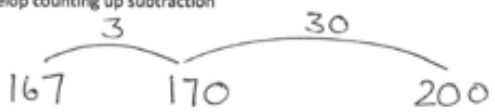
Find the difference between 122 and 67 by counting on

To jump to the next multiple of ten - reinforce the use of number bonds $122 - 67 = 55$


+3 +30 +20 +2



Develop counting up subtraction




Use counting up subtraction to find change from £1 and £10

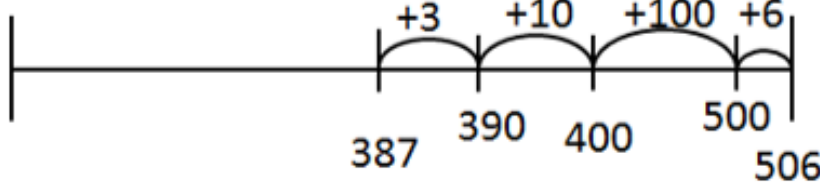


Recognise complements of any fraction to 1, e.g. $1 - \frac{1}{4} = \frac{3}{4}$ or $1 - \frac{1}{2} = \frac{1}{2}$


Counting up


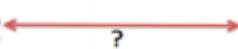
Find a difference between two numbers by counting up from the smaller to the larger, e.g. $506 - 387$



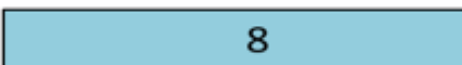




The difference between 387 and 506

Peter  8 -


Jenny  



Model

Peter  8

Jenny  


Model

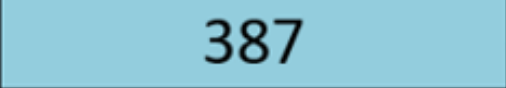
Peter  8

Jenny  

Use a number line be

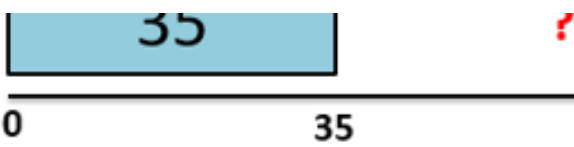
$506 - 387 =$

 506

 387

Year 4
Year 5
Year 6

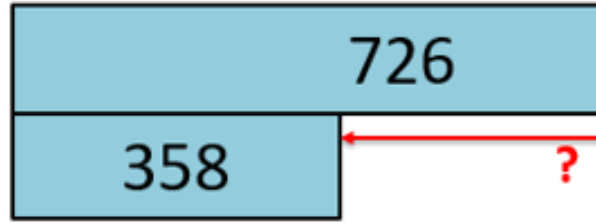
50 + 4



Expanded column subtraction

$$\begin{array}{r}
 600 \quad 110 \quad 16 \\
 \cancel{700} \quad \cancel{20} \quad \cancel{8} \\
 - 300 \quad 50 \quad 8 \\
 \hline
 300 \quad 60 \quad 8
 \end{array}$$

Expanded column subtraction with exchanging.



Begin to use compact column subtraction

$$\begin{array}{r}
 6 \quad 11 \quad 16 \\
 \cancel{7} \quad \cancel{2} \quad \cancel{8} \\
 - 3 \quad 5 \quad 8 \\
 \hline
 3 \quad 6 \quad 8
 \end{array}$$

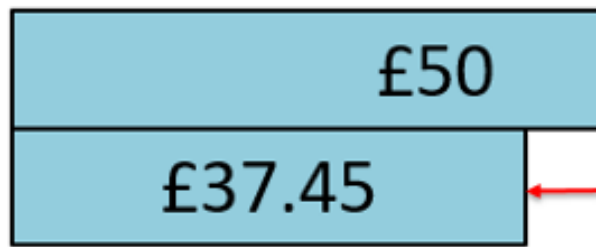
Peter brought a coat for £37.45. He paid £50. How much change will he get?

Use counting up subtraction to find change from £10, £20, £50 and £100

£34.75 £35 £40 £50

25p £5 £10

Subtract like fractions, e.g. $\frac{1}{8} - \frac{1}{8} = \frac{2}{8}$



Compact column subtraction for numbers with up to 5 digits

$$\begin{array}{r}
 0 \quad 15 \quad 13 \quad 1 \quad 14 \\
 \cancel{1} \quad \cancel{0} \quad \cancel{2} \quad \cancel{7} \quad \cancel{0} \\
 \hline
 \end{array}$$

- Key Vocabulary
- subtract
 - minus
 - less than
 - number

All children must be able to use the methods before moving on to other

Find doubles to double 50 using partitioning.
Use doubling as a strategy in multiplying by 2.
e.g. 18×2 is double 18 (36).

$$\begin{array}{r} 48 \\ 80 \\ + \\ 16 \\ \hline 96 \end{array}$$

Grouping

Recognise that multiplication is commutative, e.g. $4 \times 8 = 8 \times 4$.
Multiply multiples of 10 by single-digit numbers, e.g. $30 \times 8 = 240$.
Multiply friendly 2-digit numbers by single-digit numbers, e.g. 13×4 .

Using number facts

Know doubles to 20 and doubles of multiples of 5 to 100, e.g. double 45 is 90.
Know doubles of multiples of 5 to 100, e.g. double 85 is 170.
Know 2x, 3x, 4x, 5x, 8x, 10x tables facts.

Use arrays:



$5 \times 3 = 15$

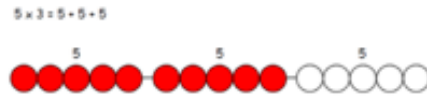
$5 \times 3 = 3 + 3 + 3 + 3 = 15$

$3 \times 5 = 5 + 5 + 5 = 15$

$3 \times 5 = 15$

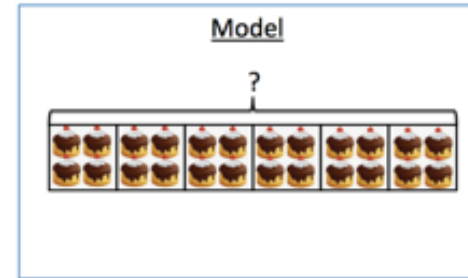
Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times _ = 6$.

Use practical apparatus:



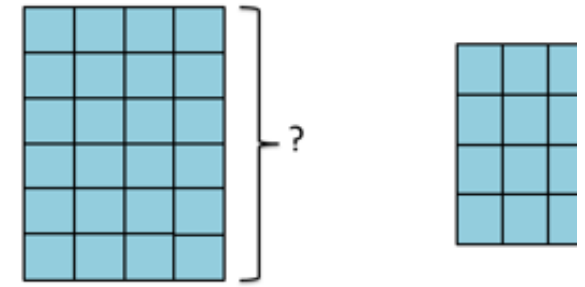
Use Numicon and counters to create arrays.

Pictorial bar model



Calculate

$6 \times 4 =$



$6 \times 4 = 24$

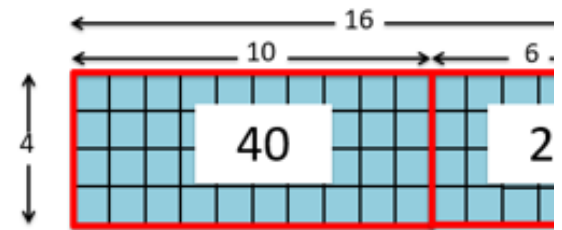
$4 \times 6 =$

Introduce the **grid method** for multiplying 2-digit by single digit numbers.
Start by repeating addition alongside array.

Use concrete resources to show grid method.
E.g. counters

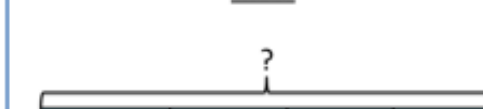
| | | |
|---|-------------|----|
| x | 10 | 6 |
| 4 | 10 | 6 |
| | 10 | 6 |
| | 10 | 6 |
| | 40 | 24 |
| | = 64 | |

| | | |
|---|-------------|----|
| x | 10 | 6 |
| 4 | 40 | 24 |
| | = 64 | |



In one bag there are 23 sweets.
How many are there in 4 bags?

Model



Build on partitioning to develop grid multiplication.

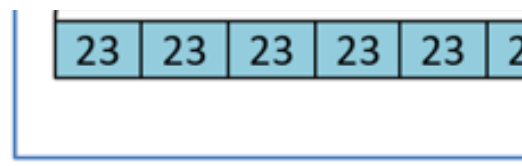
Use grid multiplication to multiply 3-digit by 1-digit numbers.

| | | | |
|---|----|----|----|
| | 10 | 10 | 4 |
| 4 | 40 | 40 | 12 |

Year 4

$$\begin{array}{r} 720 \\ + 108 \\ \hline = 828 \end{array}$$

$$\begin{array}{r} 140 \\ \times 7 \\ \hline 161 \end{array}$$



Year 5

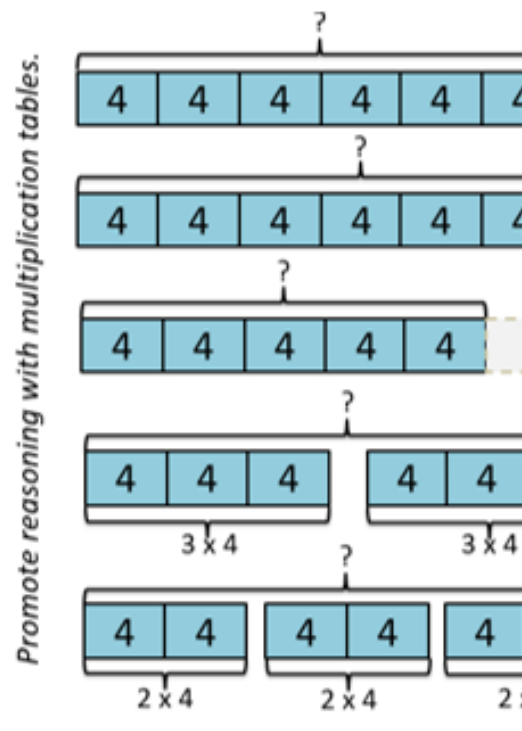
Use column multiplication (ladder) to multiply 3-digit by 1-digit.

346×9 is approximately $350 \times 10 = 3,500$

$$\begin{array}{r} 346 \\ \times 9 \\ \hline 54 \quad 6 \times 9 \\ 360 \quad 40 \times 9 \\ 2700 \quad 300 \times 9 \\ \hline 3114 \end{array}$$

Bar modelling and reasoning:

- "If I know, I also know..."
- Can be applied to higher numbers



Key vocabulary:

- multiply
- times by
- lots of

Year 6

4346×8 is approximately $4500 \times 10 = 45,000$

$$\begin{array}{r} 4346 \\ \times 8 \\ \hline 48 \quad 6 \times 8 \\ 320 \quad 40 \times 8 \\ 2,400 \quad 300 \times 8 \\ 32,000 \quad 4000 \times 8 \\ \hline 34,768 \end{array}$$

Extend to simple decimal numbers with one decimal place.
 Know that decimals should be lined up correctly based on their place value.
 Know that $4.9 \times 3 = (49 \times 3) \div 10$.

Use grid method as default, then understanding of $\div 10$.

Long multiplication 2 digit by 2 digit

72×38 is approximately $70 \times 40 = 2800$.

$$\begin{array}{r} 72 \\ \times 38 \\ \hline 72 \times 30 \quad 2160 \end{array}$$

$$\begin{array}{r} 258 \\ \times 16 \\ \hline 2580 \\ 1548 \\ \hline 4128 \end{array}$$

Children must be able to use these methods before moving on to other strategies

0 5 10 15 20

Doubling and halving
 Find half of even numbers to 100 using partitioning.
 Use halving as a strategy in dividing by 2.
 e.g. $36 \div 2$ is half of 36.

$$\begin{array}{c} 36 \\ / \quad \backslash \\ 15 \quad \quad 3 \\ + \quad \quad \quad \\ \hline 18 \end{array}$$

Grouping
 Recognise that division is not commutative, e.g. $16 \div 8$ does not equal $8 \div 16$.
 Relate division to multiplications 'with holes in', e.g. $\square \times 5 = 30$ is the same calculation as $30 \div 5 = ?$ thus we can count in 5s to find the answer.
 Divide multiples of 10 by single-digit numbers, e.g. $240 \div 8 = 30$.

Using number facts
 Know halves of even numbers to 40.
 Know halves of multiples of 10 to 200, e.g. half of 170 is 85.
 Know 2x, 3x, 4x, 5x, 8x, 10x division facts.
 Use division facts to find unit and simple non-unit fractions of amounts within the times tables, e.g. $\frac{3}{4}$ of 48 is $3 \times (48 \div 4)$.

- Use a question mark to show what is missing

28

Using bar model to visualise halving

48

Question mark you are working on

Using number facts
 Know times tables up to 12×12 and all related division facts.
 Use division facts to find unit and non-unit fractions of amounts within the times tables, e.g. $\frac{7}{8}$ of 56 is $7 \times (56 \div 8)$.

Know and understand sharing and grouping:

Children should be taught to recognise whether problems require sharing or grouping.

Sharing:
 Jenny has 30 cakes. She wants to share them equally into 5 boxes. How many cakes should go in each box?

Grouping:
 Jenny has 30 cakes. She wants to pack them into boxes, with 5 cakes in each box. How many boxes will she need to pack all the cakes?

30

Use a bar to show sharing

30

Work out needed by are hiding

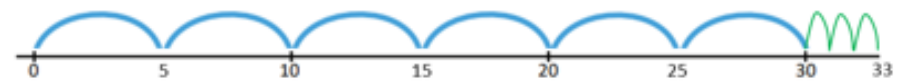
Arrays

This represents $12 \div 3$, posed as 'how many 3s are hiding in 12?'

12

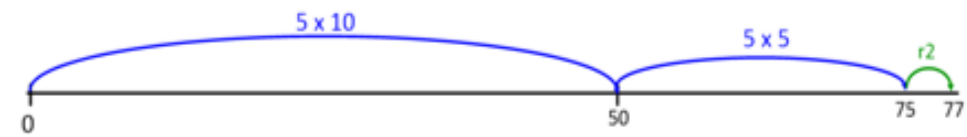
Arrays are bar model

Year 4

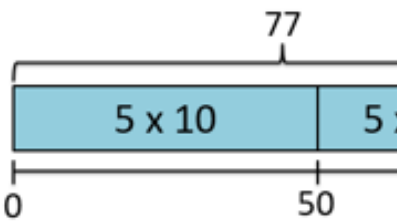


Leading to:

$77 \div 5 =$



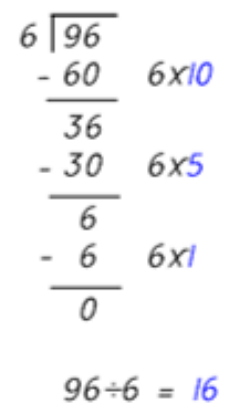
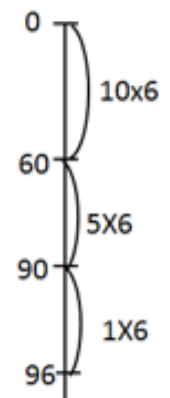
= 15 groups of 5 and 2 leftover
= 15 remainder 2



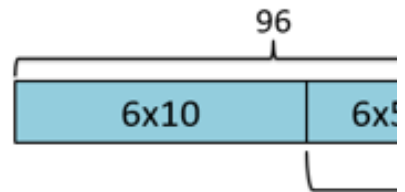
Year 5

Chunking method: Support by using a vertical number line as a bridge between methods.

$96 \div 6 = 16$

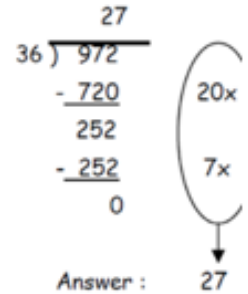


Must be aligned in place value for subtracting.



Year 6

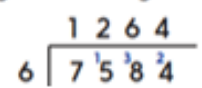
Introduce **long division by chunking** for dividing by 2 digits:



- Find out 'How many 36s are hiding in 972?' by subtracting 'chunks' of 36 until zero is reached (or until there is a remainder).
- Teach pupils to write a 'useful' list first at the side that will help them decide what chunks to use. E.g. 'Useful' list: $1x = 36$
 $10x = 360$
 $100x = 3600$
- Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots? Can we use 100 lots?' As children become confident with the process, encourage more efficient chunks to get the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' list.

Extend to decimals with up to two decimal places. It is much easier for pupils to convert the decimal to a whole number and then convert the answer back to a decimal.

Short division of 3-digit and 4-digit numbers by single-digit numbers.



Short division, for dividing by a single digit: e.g. $6497 \div 8$

Short division with remainders. Pupils should continue to use this method, but with numbers of at least 4 digits and remainders.